Electrical conductivity of low-pressure shock-ionized argon

By P. R. SMY AND H. S. DRIVER
Physics Department, University of British Columbia, Vancouver 8, B.C., Canada

(Received 6 October 1962 and in revised form 30 April 1963)

The electrical conductivity of shock-ionized argon produced in an electromagnetic shock tube of low attenuation has been measured at shock speeds of Mach 10–33, with initial pressures of 0·01–2·0 mm Hg. These measurements extend considerably the range of previous measurements performed with pressure-driven shock tubes. With the higher initial pressures or at the highest Mach numbers the measured conductivity is in good agreement with the previous measurements and with the Spitzer–Harm (1953) formula for the conductivity of a fully ionized gas. With the lower initial pressures (which have not previously been investigated) and at the lower Mach numbers the conductivity falls to less than half of the Spitzer–Harm value. Order-of-magnitude calculations show that diffusion of atoms, and heat conduction by the plasma atoms from the plasma to the shock-tube walls, can cause appreciable plasma cooling (and hence a reduction of the electrical conductivity) with the lowest initial pressures. This mechanism in conjunction with non-attainment of equilibrium ionization appears to explain the observed diminution in conductivity at the lowest pressures, but not the reduced conductivity at the medium pressures.

Induced e.m.f. flow-velocity measurements indicate steady-flow conditions in the shock tube while photomultiplier measurements of the plasma radiation indicate that the column of shock-heated gas is 10–20 cm long; this latter figure is supported by the conductivity measurements. The fact that the length of the shock-heated gas column is not drastically shortened at low initial pressures in contrast to the work of Duff (1959), Roshko (1960) and Hooker (1961) is attributed to the fact that in this experiment both driver and driven gases are at high temperature.

Introduction

This paper reports measurements of the electrical conductivity of shock-ionized argon for initial downstream gas pressures of 0·01–2 mm Hg and shock velocities of Mach 8–33. These ranges of initial pressure and shock velocity were obtained with a low-attenuation electromagnetic shock tube (Smy 1962) and extend similar measurements performed at higher initial pressures and lower shock velocities by Lin, Resler & Kantrowitz (1955), Pain & Smy (1960), and Patrick & Brogan (1959). All three groups of workers used a search coil to measure the strength of currents induced in shock-ionized argon as it moved through an external magnetic field. The same technique has been used by Lamb & Lin (1957) to measure the conductivity of shock-ionized air.

An electromagnetic shock tube of low attenuation was used in this experiment for several reasons:

(1) It was possible to obtain a range of shock speeds and initial pressures outside those used with previous argon conductivity measurements. [Lin *et al.* 1955, Mach number 7–17, initial pressure 1–100 mm Hg; Pain & Smy 1960, Mach number 7–22, initial pressure 0·1–10 mm Hg.]

(2) The low attenuation of the shock wave enabled the necessarily delicate induced-current measurements to be made in the absence of driver electric and magnetic fields.

(3) Work by Duff (1959), Roshko (1960) and Hooker (1961) showed that at initial pressures less than 1 mm Hg boundary-layer effects can drastically reduce the length of the column of shock-heated gas. Measurements by Hooker ($M < 5$) using a pressure-driven shock tube verified the theory at these shock speeds. The implications of this theory, if it is applicable to electromagnetic shock tubes, are dire indeed. For this reason it was hoped to obtain some approximate values for the length of the ionized gas column from the conductivity measurements and so test the applicability of the theory to electromagnetic shock tubes.

(4) All previous conductivity measurements of this kind had been performed with pressure-driven shock tubes. By comparing the measurements obtained with this electromagnetic shock tube with previous measurements (where conditions were the same) it was hoped to determine if pressure and electromagnetically driven shock tubes produced identical plasmas for a given shock strength and initial pressure.

The conductivity of argon at low pressures is of interest for several reasons. Previous measurements by Pain & Smy (1960) indicated that at the lowest initial pressures in their experiment (0·1 mm Hg) the plasma conductivity fell below the theoretical value given by the formula of Spitzer & Harm (1953) whereas at higher pressures agreement between the two quantities was good. By extending the initial pressure range downwards it was hoped to obtain more information about this discrepancy between theory and experiment.

The shock tube is a useful tool for studying magnetohydrodynamic interactions (Dolder & Hide 1958). The 'strength' of a magnetic interaction is given by the value of the parameter

$$\Omega = \sigma B^2 L/\rho u$$

where ρ is the plasma density, u the plasma velocity, σ the plasma conductivity, B the applied field strength and L a characteristic length. It can be seen that other quantities remaining constant, the strongest interactions will be obtained with the lowest densities (initial pressures). Thus the conductivity of shock-heated plasma with low initial pressures has considerable significance in magnetic interaction work.

The results given later in this paper show that the plasma conductivity is often considerably less than that predicted by the Spitzer–Harm formula. This is partially attributed to cooling of the shock-heated plasma and for this reason the subject of heat loss will be considered in some detail.

Heat losses from the shock-heated plasma

Any loss of heat from the shock-heated plasma will lead to a decrease in plasma temperature behind the shock front. Petschek *et al.* (1955) assume that continuum radiation is the predominant heat-loss mechanism and apply the Unsold–Kramers formula for continuum radiation to obtain a characteristic cooling time

$$\tau = 6 \times 10^8 (kT_{\text{ion}})(kT)^{\frac{1}{2}} hc^3 m^{\frac{3}{2}}/e^6 N_e \tag{1}$$

where k is Boltzmann's constant, T_{ion} is the ionization potential expressed in °K, T the plasma temperature, N_e the electron density, e the electronic charge and m the electronic mass.

They obtained spectroscopic measurements of the plasma radiation that were in accord with values predicted by the Unsold–Kramers theory, and moreover found the cooling time given by the above formula to be in good agreement with the cooling time measured by the decay of plasma radiation and the decay of electrical conductivity. In these experiments the working gas was argon and the lowest initial pressure used was 1 mm Hg.

It can be seen in equation (1) that cooling due to continuum radiation will be less important with low initial pressures (i.e. low N_e) since $\tau \propto 1/N_e$. This trend in cooling times is opposite to the trend implied by the low-pressure results of Pain & Smy (1960) if we attribute a diminished plasma conductivity to plasma cooling. Certainly, cooling times calculated from equation (1) for initial pressures less than 1 mm appear to be much too large to affect plasma conductivity in the region of the shock front. Considering these facts it seems logical to consider some form of heat loss, other than that of continuum radiation, which will predominate at initial pressures of less than 1 mm Hg and will give cooling times that decrease with decreasing initial pressure.

The three heat-loss mechanisms which will be discussed become important when the mean free path of atoms in the plasma becomes an appreciable fraction of the shock tube diameter. For the majority of measurements given in this paper ($M < 30$) the plasma is less than 50 % ionized and so consists mainly of neutral atoms. The atom-ion collision cross-section will approximately equal the atom-atom collision cross-section ($\sim 10^{-15}\,\text{cm}^2$). The atom-electron collision cross-section will be somewhat less than this ($\sim 10^{-16}\,\text{cm}^2$). Atoms will collide more frequently with electrons than ions or other atoms due to the greater electron velocity, but this will be more than offset by the much smaller momentum associated with the electrons. Thus the atoms behave as if they are in a gas of neutral atoms and we can obtain at least an order-of-magnitude estimate of the mean free path of atoms in the plasma by reference to the normal mean-free-path pressure tables. If we assume a density ratio across the shock of $R = 4$ then we obtain for an initial pressure range 0·01–1 mm Hg a mean free path $l = 2 \times 10^{-1}$ to 2×10^{-3} cm.

The average 'age' of plasma within one shock-tube diameter (L cm) of the shock front is

$$t \sim \tfrac{1}{2} LR/u_1, \tag{2}$$

where u_1 is the shock velocity. This 'age' is of some importance since with many types of plasma investigation the finite time or space resolution of the diagnostic method demands a minimum length of plasma of a few cm. For instance, the spatial resolution of plasma conductivity is limited to a not very small fraction of the shock-tube (search coil) diameter. In the experiments described by Petschek *et al.* (1955) it was found the time resolution of the spectroscopic equipment was $\sim 3\,\mu$sec, which with a shock velocity of Mach 20 implies a spatial resolution of 1·5 cm. Thus in any calculation of heat loss (particularly in conjunction with this type of conductivity measurement) we retain considerable experimental significance if we consider the plasma contained in the volume extending one shock-tube diameter back from the shock front to be in thermal equilibrium at a single reduced temperature due to heat losses from the plasma.

We will first consider the effect of thermal conduction. The thermal conductivity λ of a gas in terms of its density ρ, mean free path l, mean particle velocity $\bar{v}$ and specific heat c_v is given by (Chapman & Cowling 1952)

$$\lambda = \tfrac{1}{2}\rho l \bar{v} c_v. \tag{3}$$

By applying the heat-conduction equation to a sample of plasma whose physical dimensions are roughly L we obtain a cooling time τ where

$$\tau \sim \tfrac{1}{4}L^2/l\bar{v}; \tag{4}$$

if ΔT is the fall in temperature in time t then

$$\Delta T = 4l\bar{v}tT/L^2. \tag{5}$$

The second heat-loss mechanism to be considered is that due to the diffusion of hot atoms to the cold shock-tube walls. The diffusion coefficient D of atoms in the plasma defined by the equation $D\nabla^2 n = \partial n/\partial t$ (where n = number density of atoms and ions) is given approximately by the relation (Kennard 1938)

$$D \sim \tfrac{1}{2}\bar{v}l. \tag{6}$$

This mechanism gives a cooling time τ_D which again is given by

$$\tau_D \sim \tfrac{1}{4}L^2/l\bar{v}. \tag{7}$$

In this calculation we have merely considered the heat loss due to the diffusion of plasma atoms to the walls of the shock tube, but it should be noted that the mechanism of ambipolar diffusion might also give a cooling time of the same order of magnitude as τ_D in equation (7).

The third cooling mechanism to be considered is that of diffusion of the colder atoms of the driver gas forward into the shock-heated gas. If the total length of the shock-heated column is not much greater than L, the shock-tube diameter, and the driver-gas atomic weight is roughly equal to the shock-heated-gas atomic weight then a very approximate cooling time due to this third mechanism is

$$\tau \sim \tfrac{1}{4}L^2/l\bar{v}.$$

The important problem of driver-gas diffusion in an electromagnetic shock tube is a very complex one and it must be emphasized that the cooling-time relation given above is only an order-of-magnitude calculation.

In a very approximate manner we have considered the heat loss due to three mechanisms, each of which results in a cooling time of $\tau \simeq \frac{1}{4}L^2/l\bar{v}$. The combined heat loss due to these mechanisms will result in a cooling time

$$\tau_t \sim L^2/12l\bar{v} \tag{8}$$

and a consequent fall in temperature ΔT_t of

$$\Delta T_t \simeq 12l\bar{v}Tt/L^2 = 12l\bar{v}TR/Lu_1.$$

With $L = 5\,\text{cm}$, $u_1 = 5\times10^5\,\text{cm/sec}$, we obtain $\Delta T_t/T \simeq 0{\cdot}4$, $0{\cdot}04$ and $0{\cdot}004$ with initial pressures of 0·01, 0·1 and 1 mm Hg. For a given initial pressure the product Rl is constant and the ratio $\bar{v}/u_1$ varies slowly with shock velocity, and so bearing in mind the approximate nature of these figures they should apply over the experimental range of shock velocities given in this paper. Since in the expression for electrical conductivity σ given by Spitzer & Harm (1953)

$$\sigma = \frac{10^5}{6{\cdot}53}\frac{T^{\frac{3}{2}}}{\log\Delta}, \tag{9}$$

$\log\Delta$ varies slowly with initial gas pressure or temperature then $\sigma \propto T^{\frac{3}{2}}$ approximately and so the consequent fractional reduction in conductivity $\Delta\sigma/\sigma \simeq 0{\cdot}6$, 0·06 and 0·006 with initial pressures of 0·01, 0·1 and 1 mm Hg.

The main conclusion to be drawn from these order-of-magnitude calculations is that the electrical conductivity may fall substantially below the Spitzer–Harm value with initial pressures of 0·1 mm Hg and below.

Other factors affecting the length of the ionized gas column

An important factor in shock-tube performance is the length of the column of shock-heated gas produced by the shock tube. If the density ratio across the shock wave is R (> 4 in our experiments) then we would expect that the length of shock-heated gas driven past a point a distance x from the driver section would be x/R. As stated above, theoretical and experimental studies by Duff (1959), Roshko (1960) and Hooker (1961) show that this is certainly not so at Mach numbers < 10 and pressures $< 4\,\text{mm Hg}$. While these findings have very great significance in pressure- and combustion-drive shock-tube research it is questionable whether the results are directly applicable to electromagnetic shock tubes. Electromagnetic shock tubes operate at shock speeds much in excess of the maximum shock speed at which the theory has so far been verified; also the conditions in the driver gas immediately behind the contact surface are very different in the two types of shock tube.

In magnetic interaction work the significant factor is not the length of the column of shock-heated gas but the length of the column of electrically conducting plasma. That these lengths are not necessarily the same has been shown by the experiments of Petschek & Byron (1957), who measured the time required to obtain equilibrium ionization behind the shock front. They found that this time decreased with increasing Mach number and was inversely proportional to initial pressure. The quantity $P\tau'$, where P = initial pressure (cm Hg) and τ' = relaxation time (μsec), varied from 0·2 at Mach 20 to 100 at Mach 10. The ion-density profile between the shock front and the equilibrium ionization front was reasonably linear.

Experimental method

The low-attenuation electromagnetic shock tube used in this experiment differs from other electromagnetic shock tubes in that the driver section contains air (or nitrogen) at about atmospheric pressure and is separated from the rest of the shock tube by a thin diaphragm. The shock tube is energized by a 50 μF condenser bank charge to voltages of up to 20 kV and produces shock speeds up to 10^6 cm/sec with initial pressure below 1 mm Hg in argon or 1 cm Hg in hydrogen.

The shock tube was well suited to this experiment in that low downstream gas pressures were attainable with high gas purity. The ultimate vacuum of the tube was substantially less than 1 μ and the leak rate of the system was less than 10 μ/h. In practice the experiment was conducted with a continuous flow of argon which on average replenished the gas in the shock tube every 10 sec.

Two photomultipliers actuated by the plasma luminosity recorded the velocity of the shock wave. The first photomultiplier upstream of the test section triggered the oscilloscope time base, the second photomultiplier somewhat downstream of the test section monitored the luminosity of the oscilloscope trace and so provided a record of shock velocity on each search-coil voltage wave-form. The driver-section–test-section separation was 1 metre.

The configuration of the external magnetic field was of the double opposed-coil type as described by Pain & Smy (1960). The calibration and interpretation of the search-coil response was accomplished in the way described by them.

In brief, the instantaneous e.m.f., V, of the induced current measuring search coil is given by an expression

$$V = u_1^2 \sigma k(z)\, \partial B_z/\partial z,$$

where z is the position of the front of a semi-infinite conducting column moving along the z-axis with velocity u_1 into an inhomogeneous field (this formula is only correct at low magnetic Reynolds number); $k(z)$ is measured in a separate calibration experiment with a metal shim of known conductivity. The process of search-coil calibration and the interpretation of search-coil voltage waveforms were discussed in detail previously. As a check on the experimental value of $k(z)$, a theoretical calculation was also carried out giving a value in agreement with the experimental value.

The external field current was d.c. and was obtained from a large storage battery. The resulting field strength of about 100 G was sufficient to ensure adequate search-coil response but was very much below the threshold for appreciable interaction with the plasma flow ($\simeq 10^3$ G). Apart from the complex and to some extent unknown conditions arising from a strong magnetic interaction (joule heating and irregular plasma flow) the situation is also complicated with the onset of non-scalar conductivity (see Schluter 1950). Thus for interpretive as well as experimental simplicity the external field strength was limited to the low value given above.

Since many electromagnetic shock tubes suffer from irregular and irreproducible flow conditions behind the shock it was decided to measure the flow velocity behind the shock as a function of time. In this experiment the flow velocity was measured by recording the e.m.f. of an open-circuit M.H.D. generator as done by Sakuntala *et al.* (1959). The generator consisted of two small electrodes situated at opposite ends of a shock-tube diameter. A magnetic field was applied in a direction perpendicular to both the electrode axis and the flow direction, and induced an e.m.f. between the electrodes of $V = uxB_A$ (where u = flow velocity of interelectrode gas, B_A = applied field strength, x = electrode separation) between the electrodes. The induced voltage was recorded on a double-beam oscilloscope and thus gave a direct measure of the flow velocity averaged across a shock-tube diameter. The other beam of the oscilloscope recorded the

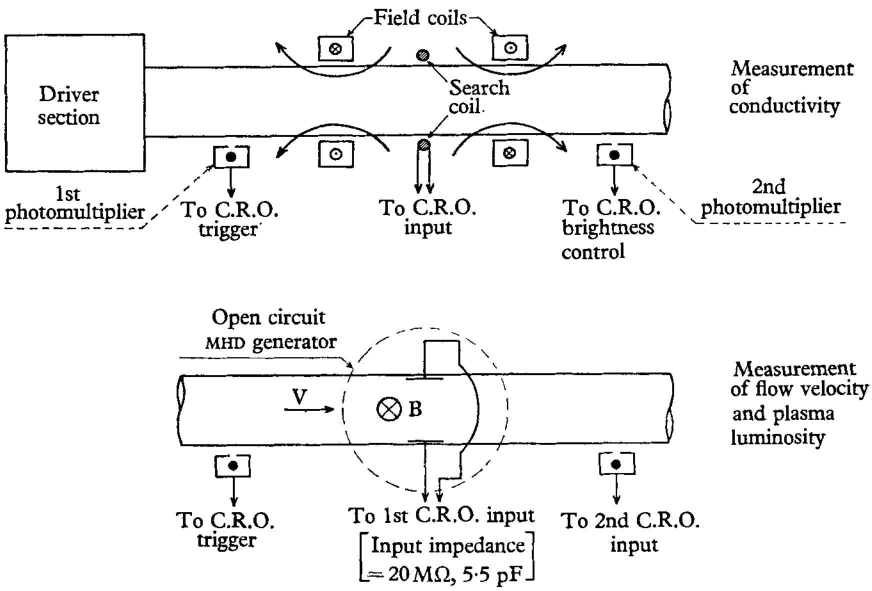

FIGURE 1. Experimental configurations for measurement of plasma conductivity, flow velocity and luminosity.

e.m.f. of the second photomultiplier. These latter measurements of flow speed and photomultiplier e.m.f. were not made at the same time as the conductivity measurements, because of the danger of interference between the two applied magnetic fields. The experimental layout for all of these measurements is shown in figure 1.

Discussion of experimental results

The measured plasma conductivities are given in figure 2 as a function of initial downstream pressure and shock Mach number. Also shown are the theoretical conductivities calculated from the Spitzer–Harm formula (modified by Lin *et al.* 1955) and the results of previous experimental work where applicable. The theoretical values are taken where possible from the results of de Leeuw (1958); otherwise the appropriate plasma temperature and electron density have been calculated directly. These theoretical values of conductivity should be accurate to within 10 %. The measured conductivities given in figure 2 are the maximum conductivities; in all cases it was found that the conductivity behind the shock rose sharply to a maximum value (in less than 2 cm) and then gradually decayed either

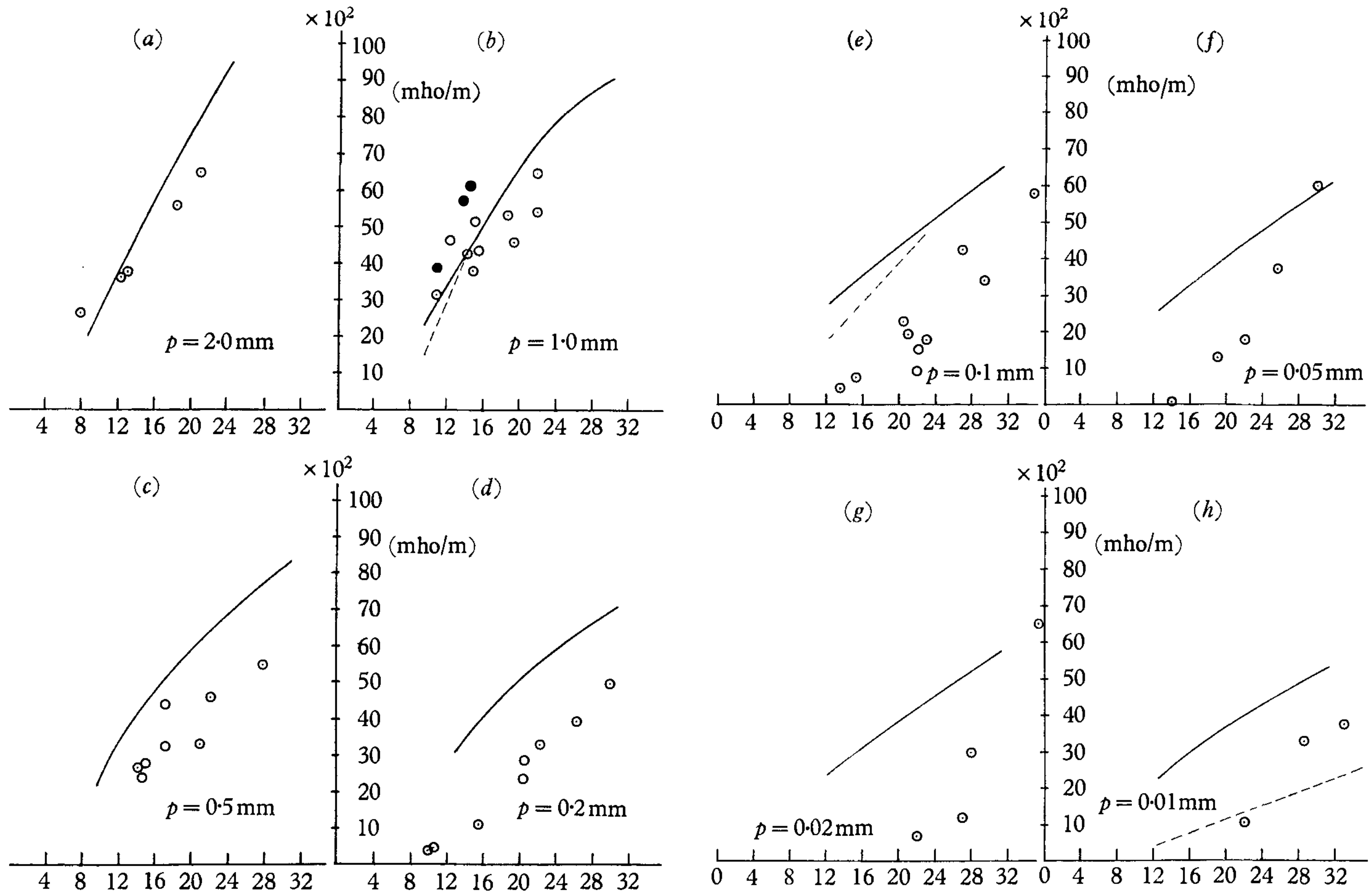

FIGURE 2. The measured and theoretical conductivities of shock-ionized argon *vs* Mach number for initial pressures of (*a*) 2 mm Hg, (*b*) 1 mm Hg, (*c*) 0·5 mm Hg, (*d*) 0·2 mm Hg, (*e*) 0·1 mm Hg, (*f*) 0·05 mm Hg, (*g*) 0·02 mm Hg, (*h*) 0·01 mm Hg. ——, Spitzer–Harm conductivity; – – – –, Spitzer–Harm conductivity for modified ionization and temperature; ●, taken from Lin *et al.* (1955); ○, taken from Pain & Smy (1960).

to zero or after a certain time fell abruptly to zero. The length of the plasma column of appreciable conductivity ($> 20\%$ of maximum) was always greater than 5 cm.

That the conductivity was that of the shock-heated argon and not of the driver gas was demonstrated by using air or hydrogen as the downstream gas: in this case the measured conductivity was reduced by a factor of at least five. With the shock tube containing air at an initial pressure of $0{\cdot}2\,\mu$ the search-coil signal was reduced by a factor of 40 and was quite unlike the usual signal.

It can be seen from the results in figure 1 that the conductivity at high pressures (1, 2 mm Hg) is in good agreement with both the Spitzer–Harm theory and with previous experimental results. At initial pressures of 0·5 mm Hg and below, the conductivity falls substantially below the Spitzer–Harm values for all but the highest Mach numbers ($M \simeq 30$) but remains in agreement with all previous measurements.

The agreement between the results presented here and the results of previous measurements performed with pressure-driven shock tubes indicates that the plasmas generated by the two types of shock tube may not be very different. In any event since the plasma conductivity is a sensitive function of temperature ($\alpha T^{\frac{3}{2}}$) it would seem that at least the temperatures of plasmas produced by the two types of shock tube are much the same for given initial conditions.

The discrepancy between the experimental and theoretical values of conductivity at low shock speeds and low initial pressures is not completely explained by either of the two mechanisms advanced above (i.e. cooling by thermal conduction, etc., or non-attainment of equilibrium ionization).

This is seen by reference to figure 2 (*b*), (*e*) and (*h*) in which the theoretical conductivities (solid line) have been reduced by an amount $\Delta\sigma/\sigma = 0{\cdot}006$, $0{\cdot}06$ and $0{\cdot}6$ to correct for cooling and then further reduced using the data of Petschek & Byron (1957) to correct for the non-attainment of equilibrium ionization (dotted line). The second correction was done in a very simple manner by averaging the calculated ionization at the centre of the plasma column over the whole column. In view of the very approximate nature of these calculations the results in figure 2 (*b*) and (*h*) (1 and 0·01 mm Hg) appear fully attributable to the two mechanisms discussed above but the experimental values in figure 2 (*e*) (0·1 mm Hg) (and also figure 2 (*d*) and (*f*)) differ considerably from the calculated values. It should also be noted that the reduction of conductivity with initial pressures $\simeq 0{\cdot}1$ mm Hg is solely due to non-attainment of equilibrium ionization ($\Delta\sigma/\sigma = 0{\cdot}06$), which should result in the conductivity reaching a maximum near the back of the plasma column whereas in fact the maximum conductivity was always achieved near the front of the column. This latter effect may be due to an enhanced plasma temperature resulting from the low ionization, but in any case it seems probable that the mechanisms of thermal conduction and non-equilibrium ionization do not completely explain the measured diminution in conductivity over the experimental range of initial pressures, although the results at the lowest initial pressures are in good agreement with the calculations.

The fact that the column of shock-ionized plasma is not drastically shortened at low initial pressures merits some discussion. Duff (1959) shows that severe boundary-layer growth at low pressures results in a larger proportion of shock-heated gas moving behind the contact surface through a thin peripheral region at the shock tube wall. For this to happen it is apparent that the shock-heated gas that 'escapes' in this way must be at a low temperature in order to occupy such a small volume. The experiments that have so far been performed which verify this theory have been conducted with pressure-driven shock tubes operated at low Mach numbers. In the measurements presented here, however, the shock speeds are in general much higher, and more important, the driver gas is itself at a higher temperature (experimental evidence for this is given later). It is thought that heat transfer from the driver gas (by radiation or conduction) to the shocked

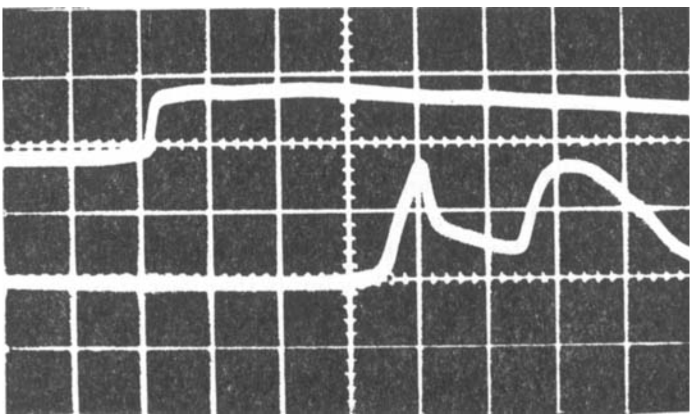

FIGURE 3. Double-beam oscillogram of plasma flow velocity (upper beam 100 V/square, B_A = 0·48 weber/m^2) and plasma luminosity (lower beam). Time is to the right (20 μsec/square).

gas near the shock-tube walls raises the temperature of the 'escaping' shocked gas and so substantially inhibits this 'escape' mechanism. This may be a property of all electromagnetic shock tubes although in this case the existence of a definite contact surface is indicated by the photomultiplier e.m.f. oscillograms.

The photomultiplier and flow-velocity measurements were recorded with one oscilloscope, and a typical double-beam oscillogram is shown in figure 3. The photomultiplier e.m.f. oscillogram shows a fairly sudden increase in luminosity which then decays slowly until a second abrupt increase occurs. The second pulse of luminosity is thought to be due to the arc-heated driver gas immediately behind the contact surface. The region of high luminosity occurring within 10 μsec of the passage of the shock front is also the region of maximum conductivity.

The open circuit M.H.D. generator e.m.f. only represents a minimum flow velocity, since the e.m.f. will be reduced by eddy currents and by the resistance of the interelectrode gas. The first of these effects was negligible since the generator field was applied over a substantial length of the shock tube, the second effect was minimized by using an attenuating circuit to the oscilloscope whose input impedance was 20 MΩ, 5·5 pF. It is found that the measured e.m.f. are in good agreement with theoretical flow velocities deduced from the measured shock velocities.

It can be seen in figure 3 that the generator e.m.f. remains constant within 20 % for several plasma flow times. While the observed fall off in e.m.f. may be due to a marked increase in interelectrode resistance or a change in flow velocity it seems very unlikely that these two mechanisms could provide a more or less constant e.m.f. for about 100 μsec over the wide range of experimental conditions. Thus it seems virtually certain that the flow velocity through the test section remains nearly constant for at least 100 μsec and thus that the flow conditions in the shock tube are similar to those in pressure-driven shock tubes. There is a difference of course in the temperature of the respective driver gases, and this is evident in both wave-forms in figure 3 since the driver gas must be somewhat conductive ($\sigma > 10^{-4}$ mho/m) in order for there to be any generator e.m.f.

The experimental results given in this paper considerably extend previous measurements of the conductivity of shock-heated argon both in the range of shock velocities and in the range of initial pressures used. Where the results do overlap previous results agreement is good and so it seems that for certain experiments this type of electromagnetic shock tube is an efficient and convenient substitute for a pressure-driven shock tube. From the point of view of research into magnetohydrodynamic interactions the results at very low pressure and at high Mach number are encouraging since they imply that very strong magnetohydrodynamic interactions are attainable under laboratory conditions with fields of the order of 1 weber/m^2.

The authors would like to acknowledge the financial support of the Atomic Energy Control Board of Canada. One of us (H. S. Driver) has carried out this work while in receipt of a Commonwealth Scholarship.

REFERENCES

Chapman, S. & Cowling, T. G. 1952 *The Mathematical Theory of Non-uniform Gases*, p. 104. Cambridge University Press.

de Leeuw, J. H. 1958 *Toronto University U.T.I.A. Rep.* no. 49.

Dolder, K. & Hide, R. 1958 *Nature, Lond.*, **181**, 1116.

Duff, R. E. 1959 *Phys. Fluids*, **2**, 207.

Hooker, W. J. 1961 *Phys. Fluids*, **4**, 1451.

Kennard, E. H. 1938 *Kinetic Theory of Gases*, p. 194. New York: McGraw-Hill.

Lamb, L. & Lin, S. C. 1957 *J. Appl. Phys.* **28**, 754.

Lin, S. C., Resler, E. L. & Kantrowitz, A. 1955 *J. Appl. Phys.* **26**, 95.

Pain, H. J. & Smy, P. R. 1960 *J. Fluid Mech.* **9**, 390.

Patrick, R. M. & Brogan, T. R. 1959 *J. Fluid Mech.* **5**, 289.

Petschek, H. E. & Byron, S. 1957 *Ann. Phys.* **1**, 270.

Petschek, H. E., Rose, P. H., Glick, H. S., Kane, A. & Kantrowitz, A. 1955 *J. Appl. Phys.* **26**, 83.

Roshko, A. 1960 *Phys. Fluids*, **3**, 835.

Sakuntala, M., Clotfelter, B. E., Edwards, W. B. & Fowler, R. G. 1959 *J. Appl. Phys.* **30**, 1669.

Schluter, A. 1950 *Z. Naturforsch.* **5***a*, 72.

Smy, P. R. 1962 *Nature, Lond.*, **193**, 969.

Spitzer, L. & Harm, R. 1953 *Phys. Rev.* **89**, 977.